fama and macbeth revisited: A Critique

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Abstract

The main conclusion of the FM study relies on the fact that the average of the slopes of 402 regressions of the monthly returns on 20 portfolios on theirs beta coefficients is positive. Considering this set of 402 slopes as a random sample drawn from the same normally distributed population, FM performed a *t*-test on the mean and conclude that the true mean significantly differs from zero. Then they took this result as a proof in favour of the theory that there is in the real world a perfect linear relationship between the expected return and the true beta of securities and portfolios or, in other terms, in favour of the theory that the market portfolio is efficient. In this article, we present several tests and arguments that put some shadow on these conclusions. In fact, several tests lead us to the conclusion that the 402 random observations above mentioned are not drawn from a normal (or symmetric stable) distribution, neither are they independent or identically distributed. Indeed, the most disturbing fact is that those observations are likely not independent.

Keywords:

CAPM, CAPT, Portfolio theory, Empirical tests, Hypothesis testing, Regression analysis, Spectral analysis, January anomaly.

JEL classification:

C12, G11, G14.

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1. Introduction

Treynor (1961), Sharpe (1964), Lintner (1965), Mossin (1966), Black (1972) and several other authors developed equilibrium models for the financial market according to which the expected return on securities can be expressed as a function of their true beta in the following manner:

$$E_i = E_0 + [E_M - E_0]\,\beta_{iM} \tag{1}$$

where E_i is the expected return on security i, E_M is the expected return on the true market portfolio M, E_0 is the expected return on a security that is riskless in the portfolio M, and β_{iM} is the true beta of security i, that is the slope of the regression of the return on security i on the return on the true market portfolio M.

Those models, that we can name Capital Asset Pricing Models or CAPMs, are the basis of theories that claim that there exist, or should exist, in the real world a perfect linear relationship between the true beta of an asset and its expected return. We will name those theories Capital Asset Pricing Theories or CAPTs.

To justify their contentions the CAPTs refer to other theories, like the efficient markets theory, as well as to other models like the random walk model. Moreover, CAPTs use plenty of arguments based on statistical analysis of empirical data. Then, in a general manner, CAPTs try to prove in one way or another that in the real world things happen, or should happen, *as if* the basic hypotheses of one CAPM or another were true.

The article of Fama and MacBeth (1973) lies within the scope of those financial market equilibrium theories. On one hand FM present theoretical arguments to justify the hypothesis of the existence of a relationship between the true beta of an asset and its expected return. On the other hand they try to show, using empirical data, the reality of such a relationship in the real world.

The objective of this article is to examine the results of the FM research to see if they really constitute a strong set of elements in favour of the theory they want to establish. All along our analyses we will adopt a pragmatic attitude. So, we will not discuss in detail or in a theoretical manner the pros and cons of the choices made by FM regarding the set of securities to be included in the study, the length of periods for the calculation of returns, the number of portfolios to be formed, the rules for the formation of those portfolios, the use of an equally weighted portfolio of NYSE stocks rather than the value-weighted market portfolio for the calculation of beta coefficients. We will focus all our attention on the statistical quality of the empirical proofs that led FM to the following conclusion: *there is on average a positive tradeoff between risk* *and return*[1]. Obviously, this is the most important point in their article. In fact, one can say that all theoretical discussion in the FM article *is for naught*[2] if the empirical proofs presented by them do not lead to this conclusion.

This article is organised as follows: In section 2 we present the empirical proof on which the main conclusion of the FM study is based. This proof is based in turn on an analysis of several averages of very many γ_{1t}. coefficients obtained from cross-sectional regressions of returns on 20 portfolios on theirs FM-betas[3]. In section 3 we present a few objections that can be raised against FM's empirical proof. We raise the question as whether we may reasonably assume that the 402 γ_{1t} coefficients calculated by FM are independent realizations of identically distributed normal (or symmetric stable) random variables. Then, in section 4, we detail our objections using the results of a few non-parametric statistical tests that were well known at the time FM published their article. These tests are: the Kolmogorov-Smirnov test, the one-proportion z-test, the Kruskal-Wallis one-way analysis of variance by ranks and the one-sample runs test[4]. The results of these tests allow us to reject one by one the hypotheses of normality, symmetry, identical distribution and randomness of the values of the γ_{1t} coefficients calculated by FM. Given the importance of these results, we decided to go a little further in our analyses. So, in section 5 we examine the empirical distribution and the time series of the γ_{1t} coefficients. In the same section, we look at the correlogram and at the power spectral density of the series of γ_{1t} values. We also present the results of the following tests: the Jarque-Bera normality test and the BDS i.i.d. test. All these analyses converge toward the same conclusion as the non parametric tests in the previous section, that is, the 402 γ_{1t} values calculated by FM are likely not realisations of normal (or symmetric stable) i.i.d. random variables. Finally in section 6 we make a summary of our findings and a conclusion.

2. Presentation of the FM empirical proof

Following a quite elaborate methodology – that we won't discuss here for the sake of clarity –, FM calculated month after month the monthly return and the FM-betas of 20 portfolios composed of NYSE securities[5]. Those calculations where performed for 402 periods of one month from January 1935 (t =1) to June 1968 (t = 402).

[1] *Cf. Fama et MacBeth (1973), p. 624.*

[2] *Cf. Fama et MacBeth (1973), p. 624.*

[3] *Fama and French (1992) use the term "FM regressions" to refer to regressions of returns on betas. Following their example, we will use the term "FM-betas" to refer to betas calculated the FM way. So, we won't confuse them with the "true betas".*

[4] *The one-proportion z-test can be found in all basic statistics textbooks. The three other tests were popularized by Siegel (1956).*

[5] *FM used the Fisher's Arithmetic Index, an equally weighted average of the returns on all stocks listed on the NYSE, to calculate the betas of the portfolios. But, from a theoretical point of view they should have used the return on the true market portfolio. In Roll (1977) there is a full discussion of this choice.*

Then, FM used the results of those calculations as input for 402 regressions. For each month t of the 1935-6/1968 period, FM ran the following cross-sectional regression[6]:

$$R_{pt}=\gamma_{0t}+\gamma_{1t}\,\beta_{pm,t-1}+\eta_{pt}, \qquad p=1,\,2,\,...,\,20, \tag{2}$$

where R_{pt} is the return on portfolio p at month t, γ_{0t} and γ_{1t} are the intercept and the slope of the regression line at month t, $\beta_{pm,t-1}$ is the FM-beta of portfolio p at the begining of month t, and η_{pt} is a random disturbance term that is assumed to have zero mean.[7]

Table A-1 in the appendix shows the month-by-month record of the least squares values of γ_{1t} in equation (2)[8].

As we can see from Table A-1, γ_{1t} values are quite variable through time and even negative in a large fraction of months. For the FM's theory, this was indeed an unexpected and disappointing fact[9].

Actually, there are 185 months out of 402 for which γ_{1t} is negative. So, one can say, that for these months there is a negative relationship between return and FM-risk. Furthermore, the proportion of positive γ_{1t} scores is not significantly different from 0.5. It is 0.5398 (=217/402) and its z-value is only 1.60. In reality, with such results, everything could stop right there.

But FM were quite resilient and perseverant. They then launch an unorthodox and creative methodology (that we will introduce shortly below) to provide evidence in favour of their theory. Unfortunately, no reference, no introductory explanation can be found in the FM's article about their unusual methodology.

Some clues can however be grasped in Fama (1976). According to Fama (1976), the hypothesis that there is a positive relationship between expected return and risk would nevertheless uphold as long as $E(\gamma_{1t})>0$, that is, as long as "on average" there is a positive relationship between return and FM-risk[10].

Let's suppose, for practical purposes, that the preceding assertion lies on solid grounds and let's continue.

[6] *Cf. FM equation (10), p. 616.*

[7] *There are two statistical problems in this regression: heteroscedasticity and cross-sectional correlation of error terms. Joint effects of those two phenomena are studied in Salazar (1986) and Salazar and Iglesias (2003). With the help of Monte-Carlo simulations, these authors proved that one can very often find "significant" and surprisingly linear relationships between observed returns and betas when in fact they do not exist.*

[8] *These values were not published in the FM article. We found them in Fama (1976).*

[9] *This is perceptible at page 624 of FM and at page 361of Fama (1976).*

[10] *Cf. Fama (1976), p. 361.*

So, in the objective of finding that there is "on average" a positive relationship between return and FM-risk, FM proceeded to calculate averages and standard deviations of the γ_{1t} coefficients for several sub-periods of different lengths. Then, in order to go along their strange methodology, they calculate *t*-statistics for these sub-periods to test the hypothesis that $\overline{\gamma}_1 = 0$. These *t*-statistics are

$$t(\overline{\gamma}_1) = \frac{\overline{\gamma}_1}{s(\overline{\gamma}_1)/\sqrt{n}} \qquad (3)$$

where n is the number of months in the sub-period, which is also the number of estimates γ_{1t} used to compute $\overline{\gamma}_1$ and $s(\gamma_1)$. Results of those calculations are presented in the table below.

● Table 1. **Average of γ_{1t} coefficients**

PERIOD	n	$\overline{\gamma}_1$	$s(\gamma_1)$	$\underline{s(\gamma_1)}$	$t(\overline{\gamma}_1)$
1935-40	72	.0109	.1161	.01368	.79
1941-45	60	.0229	.0693	.00895	2.55
1946-50	60	.0029	.0474	.00611	.49
1951-55	60	.0024	.0348	.00449	.53
1956-60	60	-.0059	.0335	.00432	-1.37
1961-6/68	90	.0143	.0483	.00509	2.81
1935-45	132	.0163	.0975	.00849	1.92
1946-55	120	.0027	.0414	.00378	.71
1956-6/68	150	.0062	.0440	.00359	1.73
1935-6/68	402	.0085	.0661	.00329	2.57

As one can see, as the length n of sub-periods increases, *t*-statistics becomes positive and, then, when = 402 , the only *t*-statistic one can calculate is equal to 2.57.

According to FM, "The small *t*-statistics for sub-periods reflect the substantial month-to-month variability of the parameters (sic) of the risk-return regressions ...[and] ... It takes the statistical power of the large sample for the overall period before values of $\overline{\gamma}_1$ that are large in practical terms also yield large *t*-values. But at least with the sample of the overall period $t(\overline{\gamma}_1)$ achieves values supportive of the conclusion that on average there is a statistically observable positive relationship between return and risk"[11].

[11] *Cf. FM (1973), p. 624.*

The value 2.57 for the t-statistic, which is calculated on the average of the 402 γ_{1t} values constitutes the essential element of the empirical proof presented by FM. In other words, it is this t-value which is the basis to state that "on average there seem to be a positive trade-off between return and risk, with risk measured from the portfolio viewpoint"[12].

3. Objections

But is the result of the t-test performed by FM as convincing as those authors seem to believe? We have doubts. The period under study is so long that is hard to believe that all necessary conditions to interpret t-statistics in the usual way are present altogether.

In a general manner, the t-test for an average requires that observed values could be considered as independent realizations of the same random variable; it may also require that this variable be normal. Now, with the exception of the normality assumption, the other conditions are not expressed in the FM article. In this section we will then review those conditions to see if they could be taken for granted.

3.1. About the normality condition

Let's say it right away, if we have 402 independent realizations of the same random variable, the shape of the underlying distribution does not matter when we want to estimate the mean or perform statistical tests on the mean of this distribution. According to the central limit theorem, the sampling distribution of the average of 402 independent and identically distributed (i.i.d.) observations is normal.

Now, FM put emphasis on the fact that the distribution of γ_{1t} coefficients as well as the distributions of returns on securities and portfolios should be normal or symmetric stable. For example in the very first page of their article, they state that "the two-parameter portfolio model [...] distributions of one-period percentage returns on all assets and portfolios are assumed to be normal or to conform to some other two-parameter member of the symmetric stable class"[13]. A few pages further, they

[12] *Cf. FM (1973), p.633.*

[13] *Except the normal distribution, all other distributions of the symmetric stable class have an infinite variance. For a brief review of the properties of symmetric stable distributions, see Fama (1971). However, it is also worthwhile to consider that Markowitz (1976) wrote concerning these distributions at page xi of his book: "Regarding the Mandelbrot-Fama contention that variance is infinite: (a) I am willing to assume that all my subjective distributions of return are bounded – e.g. between 100 percent loss and a trillion percent gain – and therefore have all their moments. (b) the strange conclusion that variance is infinite is derived by starting with the assumption that the probability distribution of hour-to-hour fluctuations in security prices has the "same form" as, say, the probability distribution of month-to-month fluctuations, which in turn has the same form as, say, the probability distribution of year-to-year fluctuations. This assumption seems less than certain when we contrast the business determinants of the year-to-year fortunes of an enterprise with the market determinants of hour-to-hour fluctuations in its stock. The assumption becomes even more questionable when we learn that the assumption implies a priori that either the distribution is normal or it has infinite variance – excluding not only the bounded distributions, but also most of the familiar unbounded distributions such as χ^2 and Student. Having assumed this much, the next step is to infer empirically that since the distribution is not precisely normal, it must have infinite variance".*

claim that "If all portfolio return distributions are to be normal (or symmetric stable), then the variables $\tilde{\eta}_{it}$, $\tilde{\gamma}_{0t}$, $\tilde{\gamma}_{1t}$, $\tilde{\gamma}_{2t}$ and $\tilde{\gamma}_{zt}$ must have a multivariate normal (or symmetric stable) distribution" [14].

Finally, FM dedicate several paragraphs to discus of (insignificant) problems related to symmetric stable distributions in the interpretation of t-tests on the mean[15].

What makes the normality issue more mysterious is the fact that several well known authors wrote, shortly before the publication of the FM article, articles that assume that the distribution of observed returns would be asymmetric. For example, Arditti (1967) took a look at the relationship between the skewness of the distribution of returns on a security and its expected return. On their side, Miller and Scholes (1972)[16] put forward the asymmetry of the distributions of returns on securities to try to explain some results they considered abnormal in the context of their CAPT.

Therefore, on one hand, FM say that distribution of γ_{1t} coefficients as well as those of returns on securities and portfolios should be normal or symmetric stable. On the other hand, there are authors who assume in their work that these distributions could be asymmetric.

This leads us to the two following questions:

1) For what reason did FM attach so much importance to the shape of the distributions of returns on securities and portfolios as well as to the shape of the distribution of γ_{1t} values?
2) Is the distribution of γ_{1t} values really normal or at least symmetric stable?

The first question is surrounded by mystery. Therefore we will just emit a simple hypothesis. It is likely that the shape of distributions has a role to play in the chain of reasoning that led FM from their own version of the CAPM toward their own CAPT[17].

Regarding the second question, we believe that the answer must be based on actual facts and not on purely theoretical considerations. So, in order to answer this question, we will perform a normality test and a symmetry test for the distribution of the γ_{1t} values, in section 4.

[14] *Cf. FM, pp. 611-612.*

[15] *Sceptical readers can look up FM, from page 619 to page 624.*

[16] *This article is cited by FM, at page 613.*

[17] *Interested readers can read Fama (1971).*

3. 2. About the i.i.d. condition

As previously mentioned, the t-test on the average requires that observed values be considered as independent realizations of the same random variable. Now, the question is whether we can consider the $402\gamma_{1t}$ values as being i.i.d..

A simple look at Table 1 brings doubts. For the period 1956-1960, the average of the γ_{1t} coefficients is -0.0059 with a t-value -1.37 and for the following period the average is +0.0143 with a t-value +2.81. It is a very astonishing shift in t-value.

Then, doubts increase if we take into account that the actual value of a γ_{1t} coefficient is influenced by the following factors:

1. the returns on the 20 portfolios formed by FM,
2. the returns on individual securities included in those portfolios,
3. the rules used by FM for the formation and periodical adjustment of the 20 portfolios, and
4. the rules FM gave themselves for the calculation of betas and portfolios returns.

Let's at this point ask two questions. How would the γ_{1t} coefficients be i.i.d. if the returns on each portfolio formed by FM were not i.i.d.? How would the return on those portfolios be i.i.d. if the returns on each security were not i.i.d.?

Considering now the returns on individual securities, it is difficult to believe that their level and volatility remained constant between 1935 and 1968. Let's think of the impact that a rise or a fall of inflation rates or interest rates could have on the expected rate of return of each individual security. Let's also think of the impact of demographic, economic, psychological and sociological changes that occur during this long period of 33 and a half years. Clearly, the returns on each individual security are not identically distributed.

Considering next the returns on the 20 portfolios formed by FM, let's think of the impact of the number of securities retained for the study on the volatility of those returns. The number of securities retained for the study went from 435 in January 1935 to 845 in January 1967; hence the number of securities included in each portfolio increased and we can therefore expect a decrease in return volatility merely because the number of securities included in each portfolio has increased.

Considering also the rules used for the calculations, let's ask ourselves a few questions. Why does the number of monthly observations used to calculate the FM-betas change in a cyclical manner every four years? This can affect randomness of γ_{1t} values. Why is the return of a delisted security excluded from the calculation of the mean return on the portfolio the delisted security belonged to? This can introduce biases in favour of the FM theory. Finally, is the number of delisted securities the same every month and the same for each portfolio?

Hence, it is clear that, at the time FM made their study, there was a bunch of reasons, good and less good, that forbade to consider the 402 γ_{1_t} values as identically distributed random variables.

Later on, some studies put in evidence recurrent phenomena in financial markets. For example French (1980) detected a "Weekend effect" and Tinic and West (1984) detected a "January effect". These studies reawaken doubts concerning the randomness of returns and consequently our doubts concerning the independence of γ_{1_t} coefficients.

But let's stop here and be pragmatic. Why not perform a few tests to answer the question? The central question is: can we consider the γ_{1_t} coefficients calculated by FM as being i.i.d. random variables?

4. Hypothesis testing

In this section, we use several techniques to test hypothesis concerning the γ_{1_t} coefficients calculated by FM. We want to know if we can reasonably consider that these coefficients:

1. have been drawn from a normal distribution,
2. have been drawn from a symmetric distribution,
3. have been drawn from identical distributions, and
4. have been drawn at random.

4.1. Testing normality: the Kolmogorov-Smirnov test

The Kolmogorov-Smirnov one-sample test is a test of goodness of fit. That is, it is concerned with the degree of agreement between the distribution of a set of sample values (observed scores) and some specified theoretical distribution. It determines whether the scores in the sample can reasonably be thought to have come from a population having the specified distribution.

We use it to test the following hypotheses:

$H_0 : \gamma_{1_t}$ scores have been drawn from a normal distribution.
$H_1 : \gamma_{1_t}$ scores have not been drawn from a normal distribution.

Let $S_N(X)$ be the observed cumulative frequency distribution of a random sample of N observations. And let $F_0(X)$ be the proportion of cases expected to have scores equal to or less than X, when X is assumed to have a normal distribution. Under the null hypothesis we would expect the differences between $F_0(X)$ and $S_N(X)$ to be small and within the limits of random errors. The Kolmogorof-Smirnof test focuses on the largest of these deviations. The largest absolute value of $F_0(X)-S_N(X)$ is called the maximum deviation D:

$$D = maximum|F_0(X) - S_N(X)| \tag{4}$$

According to our calculations[18], $D = 0.1077$. This value corresponds to the 113th observation (May 1944) whose rank is 300 when we rank observations in increasing order.

The sample distribution of D under H_0 is known. Table E in Siegel (1956) and Table 1 in Massey (1951) give certain critical values from that sampling distribution. We report some of them in the table below[19].

Table 2. Normality test

N	Critical value for D		D	Probability	Decision
	$\alpha = 0.05$	$\alpha = 0.01$			
402	0.0678	0.0813	0.1077	0.0000	H_0 Rejected

As we can see, the null hypothesis is easily rejected, provided that the set of 402 γ_{1t} values calculated by FM be considered as 402 i.i.d. realizations of a random variable. With this reserve, we can say that the distribution of the γ_{1t} coefficients is not normal.

Would it then be symmetric stable?

4.2. Testing symmetry: the one-proportion z-test

We propose here a quite simple way to test the symmetry of a distribution. It is based on a characteristic common to all symmetric distributions, whether they are symmetric stable or not. The characteristic in question is that the probability that a value is below the mean is equal to the probability that it is above the mean, and that is exactly 0.5 for a symmetric distribution.

[18] *We performed our calculations with EXCEL but the Lilliefors test of EViews gives the exact same value for D.*

[19] *According to tables of Siegel (1956) and Massey (1951), when $N>30$, one must do the following calculations: $1.36/\sqrt{402} = 0.0678$ and $1.63/\sqrt{402} = 0.0813$, for $\alpha = 0.05$ and $\alpha = 0.01$ respectively. However, Lilliefors (1967) finds those values very conservative and proposes the following calculations: $0.886/\sqrt{402} = 0.0442$ and $1.031/\sqrt{402} = 0.0514$, for $\alpha = 0.05$ and $\alpha = 0.01$ respectively.*

The one proportion z-test determines whether a sample can reasonably be thought to have come from a population having the theoretical proportion. It replaces the binomial test when the sample size n is large enough and the proportion p is not too small or too large[20].

We use this test for the following hypotheses:

H_0 : γ_{1t} scores have been drawn from a symmetric distribution.
H_1 : γ_{1t} scores have not been drawn from a symmetric distribution.

Let p be the expected proportion of γ_{1t} scores that are *smaller* than the sample mean. Under the null hypothesis, $p = 0.5$ and it is expected that the observed proportion $\hat{p}$ should be fairly close to 0.5, the expected proportion in the population.

For testing the hypothesis that $\hat{p} = p$, the z-statistic is calculated as follows:

$$z(\hat{p}) = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \tag{5}$$

where n is the number of scores in the sample. Then, if the sample is taken at random, we refer to standard normal distribution to obtain the significance of the test.

Looking at the data, we found that 225 γ_{1t} coefficients had a value *below* the sample mean which is 0.008474. The proportion $\hat{p}$ is therefore equal to $225/402$ =0.5597 and $z(\hat{p})$ is equal to 2.394. Table 3 shows the critical values for $z(\hat{p})$ at α=0.05 and α=0.01 for a two sided test.

Table 3. Symmetry test

n	$\hat{p}$	Critical value for $z(\hat{p})$		$z(\hat{p})$	Probability	Decision
		$\alpha = 0.05$	$\alpha = 0.01$			
402	0.5597	1.960	2.576	2.394	0.0166	H_0 Rejected

As one can see, we can reject quite easily the null hypothesis, under the condition, of course, that the set of 402 γ_{1t} values calculated by FM be considered as realizations of 402 i.i.d. random variables. With this reserve we can assert that the distribution of γ_{1t} coefficients is neither symmetric, nor symmetric stable.

[20] *Conditions are usually the following:* n>20, np>5, $n(1-p)$>5.

But could we at least consider those γ_{1t} values as realizations of 402 i.i.d. random variables?

4.3. Testing identical distributions:
the Kruskal-Wallis one-way analysis of variance by ranks

To test the hypothesis that all γ_{1t} coefficients really come from identically distributed populations, we chose the Kruskal-Wallis non parametric one–way analysis of variance by ranks rather than the corresponding parametric F test, in order to avoid making the assumptions concerning normality and homogeneity of variance associated with the parametric F test and to increase the generality of our findings.

Now, if the 402 γ_{1t} coefficients were issued from identically distributed populations, their distribution should be the same every year, so this distribution should be the same in 1935, 1936, ..., 1968. Also, it should be the same in January and in the other months of the year. In all cases, if distributions were identical they should have the same shape, the same mean, the same median, the same variance.

The Kruskal-Wallis one-way analysis of variance on ranks concentrates on the median of the distribution and tests the null hypothesis that k independent samples come from the same population or from identical populations with respect to their average. The test assumes that the variable under study has an underlying continuous distribution.

Let N be the total number of independent observations in the k samples. In the computation of the Kruskal-Wallis test, each of the N observations is replaced by its rank. That is, all scores from all of the k samples combined are ranked in a single series. The smallest score is replaced by 1, the next by 2, and the largest by N.

It can be shown that if the k samples actually come from the same population or from identical populations, then the H-statistic, defined below, is distributed as chi square with $k-1$ degrees of freedom, provided that the sizes of the various k samples are not too small ($n_j > 5$).

$$H = \frac{12}{N(N+1)}\sum_{j=1}^{k}\frac{R_j^2}{n_j} - 3(N+1) \qquad (6)$$

where k is the number of samples n_j is the number of observations in the j^{th} sample, $N=\sum n_j$ is the total number of observations in all samples combined, and R_j is the sum of ranks in j^{th} sample.

First test: 34 samples, one each year

In order to apply the Kruskal-Wallis test, we divided first the global sample of 402 observations into 34 samples and we tested the following hypotheses:

H_0 : γ_{1t} values are realizations from distributions with the same median.
H_1 : γ_{1t} values are realizations from distributions with different medians across years.

Results of the calculations appear in the table below.

Table 4. i.i.d. test for 34 samples

N	k	$d.f.$	$H \sim \chi^2_{33}$	α	Probability	Decision
402	34	33	53.728	0.05	0.0128	H_0 Rejected

Second test: 2 samples, one for January, the other for the others months

Then we divided the global sample of 402 observations into two samples and tested the hypotheses formulated below:

H_0 : γ_{1t} values are realizations from distributions with the same median.
H_1 : γ_{1t} values are realizations from distributions with different medians in

January and the rest of the year.

Results of the calculations appear in the table below.

Table 5. i.i.d. test for 2 samples

N	k	$d.f.$	$H \sim \chi^2_{1}$	α	Probability	Decision
402	2	1	13.492	0.05	0.0002	H_0 Rejected

As we can see in both cases (k =34 and k =2) we can reject the null hypotheses under the condition, of course, that the 402 γ_{1t} values calculated by FM can be considered as realizations of independent random variables. With this reserve, we can claim that γ_{1t} coefficients are not issued from populations with the same median.

But could we at least consider those 402 γ_{1t} values as realizations of 402 independent random variables?

4.4. Testing randomness: the one-sample runs test

If one wishes to arrive at some conclusion about a population by using the information contained in a sample from that population, then this sample must be drawn at random.

The Wald-Wolfowitz runs test checks a randomness hypothesis for a two valued data sequence. More precisely, it checks the randomness of a distribution by taking the data in the given order and marking with + the data greater than the median, and with – the data less than the median (numbers equaling the median are omitted). The technique is based in the number of runs r which a sample exhibits. A run is defined as a succession of identical symbols which are followed and preceded by different symbols or by no symbols at all.

Let n_1 be the number of + and n_z the number of −. If either n_1 or n_z is larger than 20, a good approximation to the sampling distribution of r is the normal distribution, with mean

$$\mu_r = \frac{2n_1n_2}{n_2 + n_2} + 1 \tag{7}$$

and standard deviation

$$\sigma_r = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2\,(n_1+n_2-1)}} \tag{8}$$

Therefore, when either n_1 or n_2 is larger than 20, one can test if the order of pluses and minuses occur in random order by

$$z = \frac{r - \mu_r}{\sigma_r} \tag{9}$$

The significance of any observed value of z computed from this formula may be determined by reference to the normal curve table.

We now use the one-sample runs test to test the following hypotheses:

H_0 : γ_{1t} values are drawn at random.
H_1 : γ_{1t} values are not drawn at random.

Table A-2 in the appendix shows the signs of deviations of γ_{1t} coefficients from their median. Examining this table, we see that there are 175 runs. We then proceed to make the necessary calculations and obtain the following results in the Table 6 below[21].

[21] *With MATLAB, one gets somewhat different results: $z = -2.6467$ and Prob.=0.0080. The difference is due to the fact that MATLAB uses a continuity correction which is explained in Siegel (1956), on page 140.*

Table 6. Randomness test

N	n_1	n_2	r	μ_r	σ_r	z	Probability	Decision
402	201	201	175	202	10.0125	-2.697	0.0070	H_0 Rejected

As one can see, we can easily reject the null hypothesis[22]. Consequently, we can say that it is quite unlikely that the 402 γ_{1t} coefficients would come from the realization of 402 independent random variables.

5. Examination of the distribution of γ_{1t} values

In the previous section, we applied four classical non parametric tests and we rejected one by one the hypotheses of normality, symmetry, identical distribution and randomness of the γ_{1t} values calculated by FM. Now, in this section, we make some comments and perform brief analyses in the objective of get a better understanding of the results previously obtained. We not intend to give here additional or complementary elements against the validity of FM research. We merely want to show that there are alternative ways to reach the same results as in section 4.

5.1. Signs of deviations from the median

Let's consider again the table A-2. Let's note that in the row corresponding to the year 1960 all the signs are negative. This simple fact helps to explain the results of the one-sample runs test and the Kruskal-Wallis test for the case of 34 samples. On one hand it is unlikely to observe a run of 12 times the same sign in a random sample of size 402. On the other hand, this long run extends exactly over a whole year making this year very particular. Therefore, the one-sample runs test says that the sample is probably not random and the Kruskal-Wallis test says that the γ_{1t} coefficients are not identically distributed every year.

Let's note also that there are 26 positive signs in the January column. This confirms somehow the result of the Kruskal-Wallis test for the two samples case[23]. The distribution of γ_{1t} coefficients in January is in all probability different from that of the other months. So, the Kruskal-Wallis test says that coefficients γ_{1t} are not identically distributed.

[22] *Salazar (1986) used this test with the signs of the γ_{1t} coefficients and obtained similar results: z = -2.20 with an associated probability of 0.0278. We preferred using the signs of deviations from the median to avoid ambiguities regarding the observation of February 1939 and to conform to the example given by Siegel (1956) on page 54.*

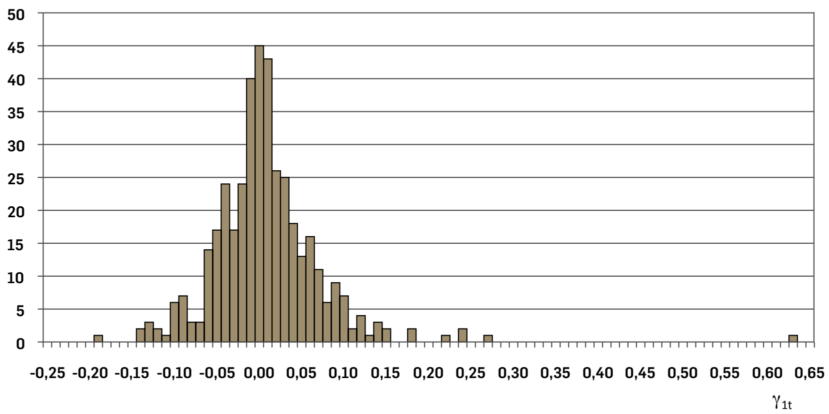
[23] *The proportion of positive signs $\hat{p}$=2634=0.7647 is significantly different from 0.5. Following the one-proportion z-test procedure, we obtain a z-value equal to 3.09, whose associated probability is 0.0020.*

Finally, note that the January effect detected in Table A-2 can be taken as a proof of lack of randomness. Nevertheless it does not help to explain the result of the one-sample runs-test. That is to say that the runs-test can't detect all types of lack of randomness.

5.2. Histogram

Figure 1 shows the histogram of γ_{1t} values. Taking a look at this histogram allows us to accept more easily the results on the non-normality and asymmetry presented in the previous section.

Figure 1. Histogram of γ_{1t}



The distribution does not look normal at all. It is asymmetric, irregular, with holes and peaks, and it contains very extreme values. In the table below we give the list of γ_{1t} coefficients that are further than 2.58 standard deviations from the mean. Each γ_{1t} coefficient is shown with its corresponding γ_{0t} coefficient.

Table 7. Extreme values of γ_{1t}

MONTH	γ_{1t}	$z(\gamma_{1t})$	γ_{0t}	$z(\gamma_{0t})$
1939-03	-0.1880	-2.97	0.0152	0.24
1943-01	0.1840	2.66	-0.0064	-0.33
1938-10	0.2235	3.26	-0.0869	-2.47
1943-02	0.2402	3.51	-0.0967	-2.73
1942-01	0.2445	3.57	-0.0768	-2.20
1938-06	0.2677	3.92	0.0405	0.91
1939-09	0.6295	9.40*	-0.2040	-5.57

In this list there is only one extreme negative observation and there are six extreme positive observations. Among these, one is 9 standard deviations away from the mean! It is the one of September 1939[24]. By coincidence, during the same month, it was registered the lowest value of the coefficient γ_{0t} for the whole period of 402 months.

5.3. Descriptive Statistics

In the table below, one finds elements that can help reach in a different way the result of the Kolmogorov–Smirnov test concerning the non-normality of the γ_{1t} coefficients. As we can see, unlike normal distribution, the empirical distribution of γ_{1t} values has a skewness coefficient somewhat different from zero and a kurtosis coefficient much higher than 3.

● Table 8. **Descriptive statistics**

STATISTIC	VALUE	STANDARD ERROR
Mean	0.008474	0.003295
Median	0.003650	0.002014**
Standard deviation	0.066055	n.a.
Kurtosis – EViews *	22.86759	n.a.
Skewness – EViews *	2.472025	n.a.

* *Based on the biased estimator for the variance*

** *Based on bootstrapping. We followed the procedure proposed by Racicot and Théoret (2001)*

Therefore it is not surprising that the Jarque-Bera normality test[25], which is constructed on those two measures, strongly rejects the normality hypothesis (JB=7021.007; Prob.= 0.0000).

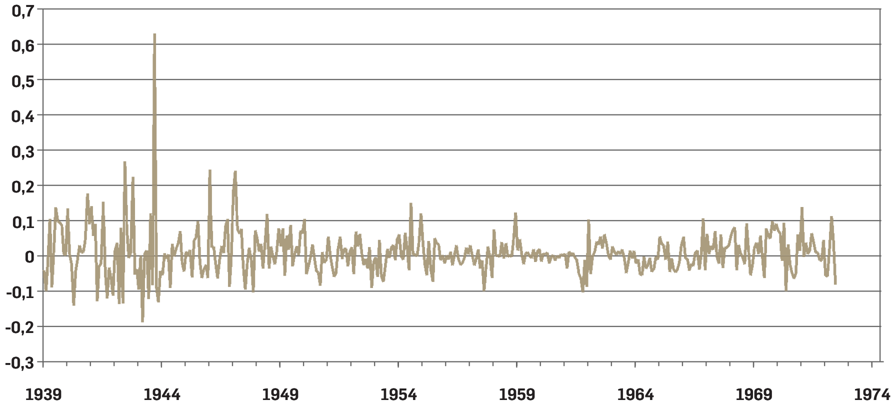
5.4. The Time Series

Figure 2 shows the evolution of γ_{1t} coefficient in function of time. This diagram doesn't help to check for randomness. However, it confirms the existence of extreme and asymmetrical values and makes us suspicious that γ_{1t} values are not identically distributed (i.d.), this time, because of differences in variances.

[24] *One considers that 2nd World War stated on September 1939. On September 1st 1939 Germany invaded Poland. A few days later the United Kingdom, Australia, New Zealand, France, South Africa and Canada declared war on Germany.*

[25] *This test is available in EViews. The original ideas are in Jarque and Bera (1987).*

Figure 2. γ_{1t} time series



We did not think it was necessary to perform a test on variances. But we decided to use the non parametric two-dimensions BDS test[26] to corroborate the fact that γ_{1t} values are probably not i.i.d.. This is a broad test. It can detect the presence of non linear and even chaotic-deterministic dependence in time series. For the γ_{1t} series, this test gives a value $z = 4.18$ that we can evaluate referring to the standard normal table. The associated probability is 0.0000 and allows us to reject the null hypothesis that γ_{1t} values are i.i.d..

But the trouble with the BDS test, as well as with other tests, is that the null hypothesis can be rejected either because the variables are not i.d., either because they are not independent, or for both reasons. However, looking at Figure 2 and considering the result of the one-sample runs test as well as the one of the Kruskal-Wallis test for the January effect, we believe that the BDS test rejects the null hypothesis for both reasons.

5.5. Correlogram

Figure 3 shows the correlogram of γ_{1t} values. The correlogram is a commonly-used tool for checking randomness in a data set. This randomness is ascertained by computing autocorrelations for data values at varying time lags. If data is random, such correlations should be near zero. If an autocorrelation is within $\pm\frac{2}{402} \simeq \pm 0.10$ it is not significantly different from zero at (approximately) 5% significance level.

[26] *This test is available in EViews. It was developed by Brock, Dechert and Scheinkmann (1986).*

Figure 3. **Correlogram**

AUTOCORRELATION	PARTIAL CORRELATION		AC	PAC	Q-stat.	Prob.
		1	0.022	0.022	0.1960	0.658
		2	0.080	0.080	2.8053	0.246
		3	-0.040	-0.044	3.4589	0.326
		4	0.038	0.033	4.0364	0.401
		5	-0.039	0.034	4.6576	0.459
		6	-0.059	-0.066	6.1004	0.412
		7	-0.028	0.017	6.4268	0.491
		8	-0.126	-0.122	13.008	0.112
		9	-0.066	0.062	14.834	0.096
		10	-0.123	-0.104	21.098	0.020
		11	0.092	0.094	24.606	0.010
		12	0.042	0.056	25.342	0.013
		13	0.038	0.008	25.938	0.017
		14	-0.024	-0.035	26.172	0.025
		15	0.157	0.138	36.459	0.002

Now, as we can see from the diagram, both correlations and partial autocorrelations significantly differ from zero for lags 8, 10 and 15. But what is more troublesome is that the highest autocorrelation (corresponding to lag 15) has a value z=0.157 $\sqrt{402}$=3.15 with an associated probability of only 0.0016.

These results raise many questions. Would the detected autocorrelations be due to cyclical phenomena on the financial market? Or, would they be due to the FM's methodology to calculate betas and returns? Hard to tell. But whatever the reason is, it raises doubts on the randomness of the data.

5.6. Power Spectral Density Estimate

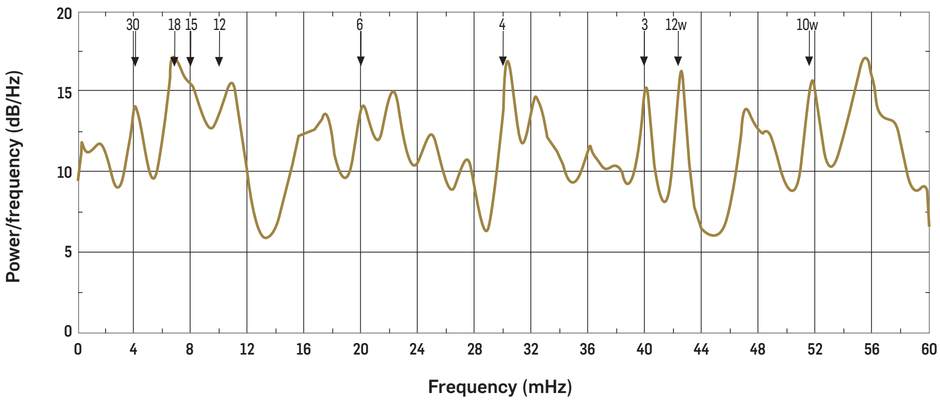
Figure 4 shows the Yule-Walker Power Spectral Density Estimate[27] of the time series of γ_{1t} values. We can detect cycles of 3, 4, 6, 12, 15, 18 and 30 months length[28] and two others of 10 and 12 weeks length. Most cycles are not extremely strong in the sense that in a time series of 402 i.i.d. values, one could see by pure chance similar cycles. But the trouble here is that several of those cycles converge towards a one year time length. This feeds our doubts concerning the randomness of the series of 402 γ_{1t} values[29].

[27] *We used MATLAB with the following commands : >> load FMg1.txt; h=spectrum.yulear(60); x=zscore(FMg1,1); psd(h,x,'Fs',0.12).*

[28] *With 120 delays the peak corresponding to 15 months becomes more and more obvious. With 240 delays, it becomes the most important.*

[29] *Important works in spectral analysis of economics series includes Granger (1966), Granger and Morgenstern (1963), Granger and Hatanaka (1964), Harvey (1975) and Wang (2003).*

Figure 4. **Yule-Walker Power Spectral Density Estimate**



6. Conclusion

Based on the tests and analysis we have carried out, we believe that the results the FM study can't be safely taken at face value. These authors did not pay attention to the validity conditions of the *t*-test they performed. They simply take these conditions for granted. This laxity constitutes a serious shortcoming in their study.

Concerning the shape of the distribution of the γ_{1t} regression coefficients calculated by FM we believe that this issue is now closed. Those coefficients are neither normal, nor symmetric stable. But, as we have mentioned, this is not the main issue in the FM article.

Most standard statistical tests depend on randomness. For the *t*-test used by FM, the validity of the conclusion is directly linked to the validity of the randomness assumption. If the researchers do not check for randomness, then the validity of their statistical conclusions becomes suspect. If the data are not random, the estimates for the parameters (such as the mean) become nonsensical and invalid.

Hence, the most important issue concerns data randomness. And we believe that we have presented enough elements that forbade us to consider the γ_{1t} regression coefficients calculated by FM as i.i.d. random variables. These coefficients are neither identically distributed, nor random.

All this is not to say that a relationship between risk and return does not exist. This means that the results of the FM research cannot be accepted in the canon of well established empirical findings. Therefore, there is no reliable evidence of a positive relationship between average returns and betas for the 1935-1968 time period.

Finally, twenty years later, Fama and French (1992) found no significant relationship between average stock returns and betas for the 1963 to 1990 time period. As Fama et French said: "The FM regressions show that market β does not help explain average stock returns for 1963-1990. In a shot straight at the heart of the SLB model, the average slope from the regressions of returns on β alone in Table III is 0.15% per month and only 0.46 standard errors from 0"[30].

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[30] *Cf. Fama and French (1992), page 438. The SLB model is the Sharpe-Lintner-Black model that we refer to in this article as Fama and MacBeth CAPT.*

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Table A1. The γ_{1t} coefficient from January 1935 to June 1968

Year	Month											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Ago	Sep	Oct	Nov	Dec
1935	-0,0413	-0,0997	-0,0126	0,1040	-0,0889	0,0105	0,1370	0,0973	0,0945	0,0869	0,0060	0,0021
1936	0,1342	0,0047	-0,0219	-0,1398	-0,0433	-0,0096	0,0292	0,0098	0,0113	0,0458	0,1763	0,0905
1937	0,1400	0,0572	0,0898	-0,1279	-0,0295	-0,0220	0,1531	-0,0449	-0,1197	-0,0333	-0,0659	-0,1122
1938	0,0144	0,0365	-0,1357	0,0786	-0,1338	0,2677	0,0746	-0,0053	0,0085	0,2235	-0,0520	-0,0380
1939	-0,0946	0,0000	-0,1880	0,0072	0,0147	-0,1213	0,1202	-0,0827	0,6295	-0,0899	-0,1312	-0,0433
1940	-0,0543	0,0069	-0,0076	0,0038	-0,0898	0,0333	-0,0010	0,0226	0,0365	0,0692	-0,0206	-0,0438
1941	0,0118	-0,0010	0,0165	-0,0609	0,0432	0,0518	0,0987	-0,0163	-0,0624	-0,0386	-0,0258	-0,0623
1942	0,2445	0,0240	0,0266	-0,0226	-0,0633	-0,0126	0,0274	0,0083	0,0859	0,1036	-0,0865	0,0001
1943	0,1840	0,2402	0,0746	0,0631	0,0759	-0,0357	-0,0950	-0,0222	0,0222	-0,0133	-0,1024	0,0720
1944	0,0495	0,0125	0,0335	-0,0348	0,0319	0,1177	-0,0355	0,0259	-0,0019	-0,0233	0,0138	0,0778
1945	0,0269	0,0770	-0,0551	0,0577	0,0174	0,0877	-0,0290	0,0037	0,0272	0,0023	0,0720	0,0633
1946	0,1036	-0,0518	-0,0303	-0,0029	0,0318	-0,0070	-0,0427	-0,0477	-0,0839	0,0104	-0,0190	-0,0116
1947	0,0543	0,0026	-0,0311	-0,0584	-0,0494	0,0173	0,0538	-0,0082	0,0188	0,0189	-0,0284	0,0132
1948	0,0249	-0,0507	0,0684	0,0272	0,0614	0,0064	-0,0256	-0,0089	-0,0350	0,0240	-0,0903	-0,0160
1949	-0,0046	-0,0582	0,0448	-0,0594	-0,0735	-0,0230	0,0190	-0,0051	0,0236	0,0312	-0,0118	0,0470
1950	0,0603	-0,0009	-0,0105	0,0646	0,0201	-0,0399	0,1501	0,0044	0,0037	-0,0001	0,0397	0,1200
1951	0,0382	-0,0239	-0,0544	0,0420	-0,0413	-0,0727	0,0498	0,0366	0,0317	-0,0126	0,0018	-0,0206
1952	0,0013	-0,0095	0,0111	-0,0267	0,0036	0,0296	-0,0075	-0,0248	-0,0199	-0,0053	0,0228	0,0054
1953	0,0291	0,0027	-0,0261	-0,0070	0,0175	-0,0353	-0,0128	-0,0993	-0,0375	0,0261	-0,0067	-0,0618
1954	0,0743	-0,0004	0,0016	-0,0026	0,0388	-0,0065	0,0359	-0,0032	0,0035	-0,0026	0,0491	0,1222
1955	0,0162	0,0463	0,0053	-0,0230	0,0059	0,0102	-0,0005	-0,0057	0,0187	-0,0164	0,0139	0,0188
1956	-0,0152	0,0056	0,0039	0,0088	-0,0207	-0,0022	0,0019	-0,0130	0,0001	-0,0062	0,0042	0,0002
1957	0,0060	-0,0346	0,0120	0,0010	0,0082	-0,0044	-0,0118	-0,0506	-0,0646	-0,1023	-0,0119	-0,0876
1958	0,1024	-0,0518	-0,0017	0,0111	0,0404	0,0317	0,0547	0,0216	0,0607	0,0361	-0,0002	-0,0096
1959	0,0260	0,0120	0,0038	0,0125	0,0054	0,0177	-0,0057	-0,0483	-0,0203	0,0145	-0,0069	0,0133
1960	-0,0205	-0,0110	-0,0520	-0,0548	-0,0030	-0,0354	-0,0262	-0,0059	-0,0440	-0,0362	-0,0019	-0,0110
1961	0,0547	0,0343	0,0291	-0,0106	0,0398	-0,0452	-0,0072	-0,0385	-0,0463	-0,0381	-0,0171	0,0295
1962	0,0538	-0,0079	-0,0087	-0,0411	-0,0226	-0,0277	0,0090	0,0151	-0,0392	0,0032	0,1051	-0,0402
1963	0,0615	0,0061	0,0274	0,0174	0,0695	-0,0041	-0,0220	0,0603	-0,0344	0,0219	0,0230	0,0313
1964	0,0558	0,0728	0,0815	-0,0394	0,0309	0,0076	-0,0049	-0,0236	0,0914	0,0122	-0,0543	-0,0276
1965	0,0231	0,0373	0,0060	0,0578	-0,0066	-0,0627	0,0773	0,0599	0,0487	0,0978	0,0718	0,0903
1966	0,0720	0,0637	-0,0139	0,0933	-0,0991	0,0317	-0,0252	-0,0486	-0,0638	-0,0500	0,0611	0,0141
1967	0,1371	-0,0027	0,0373	0,0060	0,0154	0,0640	0,0398	0,0102	0,0093	-0,0125	-0,0132	0,0445
1968	-0,0561	-0,0577	0,0146	0,1127	0,0378	-0,0809						

SOURCE: TABLE 9.3 OF FAMA (1976), *FOUNDATIONS OF FINANCE*, WHERE THEY ARE LABELED γ_{1t}.

Table A2. The γ_{1t} coefficient: signs of deviations from the median

Year	Month												Sign	
	Jan	Feb	Mar	Apr	May	Jun	Jul	Ago	Sep	Oct	Nov	Dec	+	-
1935	-	-	-	+	-	+	+	+	+	+	+	-	7	5
1936	+	+	-	-	-	-	+	+	+	+	+	+	8	4
1937	+	+	+	-	-	-	+	-	-	-	-	-	4	8
1938	+	+	-	+	-	+	+	-	+	+	-	-	7	5
1939	-	-	-	+	+	-	+	-	+	-	-	-	4	8
1940	-	+	-	+	-	+	-	+	+	+	-	-	6	6
1941	+	-	+	-	+	+	+	-	-	-	-	-	5	7
1942	+	+	+	-	-	-	+	+	+	+	-	-	7	5
1943	+	+	+	+	+	-	-	-	+	-	-	+	7	5
1944	+	+	+	-	+	+	-	+	-	-	+	+	8	4
1945	+	+	-	+	+	+	-	+	+	-	+	+	9	3
1946	+	-	-	-	+	-	-	-	-	+	-	-	3	9
1947	+	-	-	-	-	+	+	-	+	+	-	+	6	6
1948	+	-	+	+	+	+	-	-	-	+	-	-	6	6
1949	-	-	+	-	-	-	+	-	+	+	-	+	5	7
1950	+	-	-	+	+	-	+	+	+	-	+	+	8	4
1951	+	-	-	+	-	-	+	+	+	-	-	-	5	7
1952	-	-	+	-	-	+	-	-	-	-	+	+	4	8
1953	+	-	-	-	+	-	-	-	-	+	-	-	3	9
1954	+	-	-	-	+	-	+	-	-	-	+	+	5	7
1955	+	+	+	-	+	+	-	-	+	-	+	+	8	4
1956	-	+	+	+	-	-	-	-	-	-	+	-	4	8
1957	+	-	+	-	+	-	-	-	-	-	-	-	3	9
1958	+	-	-	+	+	+	+	+	+	+	-	-	8	4
1959	+	+	+	+	+	+	-	-	-	+	-	+	8	4
1960	-	-	-	-	-	-	-	-	-	-	-	-	0	12
1961	+	+	+	-	+	-	-	-	-	-	-	+	5	7
1962	+	-	-	-	-	-	+	+	-	-	+	-	4	8
1963	+	+	+	+	+	-	-	+	-	+	+	+	9	3
1964	+	+	+	-	+	+	-	-	+	+	-	-	7	5
1965	+	+	+	+	-	-	+	+	+	+	+	+	10	2
1966	+	+	-	+	-	+	-	-	-	-	+	+	6	6
1967	+	-	+	+	+	+	+	+	+	-	-	+	9	3
1968	-	-	+	+	+	-							3	3
+	26	16	18	17	19	15	16	13	17	15	13	16	201	
-	8	18	16	17	15	19	17	20	16	18	20	17		201